Topology optimization for whispering gallery mode resonator circuits based on level set expression incorporating surface effects

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Abstract

A level set-based topology optimization method for whispering gallery mode resonator circuits is presented. The effect of total internal reflection at the surfaces of dielectric disks are simulated by modeling clearly defined dielectric boundaries in the process of optimizing the topology. The electric field intensity in an optimal resonator became more than 20 times larger than the initial intensity. Dielectric structures were expressed by level set functions defined as piecewise-constant values. The expression obtained provided precise optimal configurations without any grayscale, which is the intermediate density between the dielectric materials and air. The clear dielectric boundaries of the optimal configurations were defined as iso-surfaces of the level set functions and the zero-points of the level set functions were obtained based on linear interpolations of the functions.

1. Introduction

Recent development on the fabrication of micro-nano structures has resulted in advanced optical devices such as lasers [1–5], waveguides [6], and cloaks [7,8]. Whispering gallery mode (WGM) lasers [9–13] are one of the advanced devices used to realize low-threshold laser oscillations. Light waves propagated along the surfaces of disk resonators by repeated total internal reflection. The light trajectory became circular or polygonal when WGM occurred. Hence, it is necessary to model clearly defined dielectric surfaces in simulations of WGM oscillations to incorporate the effects of reflections.

Topology optimizations [14] are powerful numerical methods for designing high performance optical devices. These optimization methods have been applied to the designs of cloaks [15], super lenses [16], and metamaterials with a negative permeability [17,18]. In the topology optimizations, some structural expressions have been proposed and topology optimizations based on homogenization [14,19] and density methods [20] are widely used for designing materials in engineering. However, these methods provide optimal configurations that include grayscale, which are the intermediate densities between the materials and air [21]. The presence of the grayscales makes it difficult to fabricate designed structures and graded optimal configurations that need discrete filtering schemes [21]. Grayscales greatly affect the device properties and the use of filtering schemes degrade the devices performances considerably when grayscales are filtered out to provide discrete configurations. To overcome the grayscale problem, a topology optimization method based on a level set expression [22] is proposed. Level set functions are defined as piecewise-constant values and become zero at structural boundaries. The level set-based topology optimization can remove the grayscale from inside the optimal configurations. However, the grayscale still exists along the structural boundaries in the optimal configurations. To design optical devices that incorporate the effect of dielectric surfaces, topology optimizations that consider clearly defined dielectric boundaries are needed.

In this work, we present a topology optimization for WGM resonator circuits based on the level set expression. Clearly defined dielectric boundaries were obtained by linear interpolations of the level set functions and the effect of the dielectric surfaces was incorporated into the optimization processes. The structures of the dielectric disk resonators were expressed as level set functions and grayscale-free optimal configurations were obtained. Optimally designed WGM resonator circuits could enhance the electric field intensity in resonators and improve the performance of devices.

2. Formulation and implementation

Figure 1(a) shows a schematic illustration of the problem for the optimization of a WGM resonators circuit. A circuit with dielectric disk resonators $\Omega_{dm}$ was designed. The resonators were designed and underwent a transformation in the design domain, $\Omega_{design}$. A fixed waveguide, $\Omega_{wg}$ guided the light waves emitted from a point source located at $(x_p,y_p)$. The width of the waveguide was $L_{wg}$. The intensity of the electric field was maximized in the $\Omega_{dm}$. A perfectly matched layer absorbing boundary condition (PML-ABC) [23] and an optimized absorbing function [24] were used to simulate light scattering in the open regions.

Figure 1(b) shows the level set functions defined on grid points and at the dielectric boundary. The zero-points of the function were obtained by linearly interpolating the function, and the iso-surfaces of the level set functions were interpreted as the dielectric boundaries. Finite elements were created based on the dielectric boundaries obtained and the grids.
can be expressed with a Helmholtz equation:

\[ \nabla^2 E + k^2 E = 0 \]

The relationship between the scattered and incident fields follows:

\[ E_{\text{sc}} = E_{\text{inc}} \]

where \( E_{\text{sc}} \) is the scattered field, \( E_{\text{inc}} \) is the incident field, and \( k \) is the wave number.

The zero-points of the level set functions were obtained by linear interpolation of the level set functions. The values of the level set functions were determined with respect to the grid points, as shown in Fig. 1(b), and were linearly interpolated at each location, having a value of 0 at the dielectric boundaries.

### 2.3. Objective functional

To design WGM resonator circuits, the total electric field \( E_z \) in the resonators \( \Omega_{\text{dm}} \) must be maximized. Hence, the objective functional for maximizing the light intensity in the resonators is defined as:

\[
F = \frac{1}{F_0} \int_{\Omega_{\text{dm}}} E_z E_z^* d\Omega, \tag{1}
\]

where \( E_z^* \) is the complex conjugate of \( E_z \), and \( F_0 \) is the integrated intensity of the total electric field for the initial configuration:

\[
F_0 = \left. \int_{\Omega_{\text{dm}}} E_z E_z^* d\Omega \right|_{\text{Initial}}. \tag{2}
\]

Topology optimizations need regularization to obtain optimal configurations because such optimizations are an ill-posed problem. Based on the formulation of the level set-based topology optimization method [22], the objective functional (1) was regularized by adding a fictitious interfacial energy term derived from the phase field model, as follows:

\[
F_t = -F + \int_{\Omega_{\text{design}}} \frac{1}{2} \tau |\nabla \phi|^2 d\Omega, \tag{3}
\]

where \( \tau \) is a positive regularization parameter that represents the ratio of the objective functional to the fictitious interfacial energy term. The above regularization with the fictitious energy term also acted as a perimeter control, a type of geometrical constraint. The geometrical constraint became weaker and the optimal configurations became more complex. For details on this geometrical constraint, please refer to the theoretical papers previously published [15,22].

### 3. Results

Figure 2 shows the initial configuration, a normalized electric field distribution, its amplitude and the corresponding objective functional value, \( F \). The value of \( F \) became 1 because the objective functional value was normalized by itself in Eq. (1).
Figure 2: (Color online) An initial configuration, electric field distribution \( E_z / |E_0^i| \) and electric field amplitude \( |E_z| / |E_0^i| \) for the initial configuration. \( E_0^i \) is the incident electric field at the center of the \( \Omega_{\text{design}} \). The relative permittivity values were \( \epsilon_{\text{dm}} = 4.0 \) and \( \epsilon_{\text{air}} = 1.0 \). The normalized frequency was \( \omega L_{\text{design}} / 2\pi \epsilon = 5.0 \).

In Figs. 2(b) and 2(c), the light waves radiated from the light source and propagated in the waveguide and a small amount of light was observed in the resonators. WGMs were not observed in the dielectric disks.

Figure 3 shows the optimal configurations obtained for \( \tau = 1 \times 10^{-6} \) and \( 1 \times 10^{-7} \), and the distributions of the total electric field and its amplitude for the optimal configuration. The \( F \)'s for the optimal configuration reached values roughly 20 times higher than the initial configuration. A large structural difference was not observed between the initial and the optimal configurations, as shown in Figs. 3(a) and 3(b). In Figs. 3(c) to 3(f), “circuits” of light trajectories along the disk surfaces and the waveguides were observed.

When the optimal configurations were compared with the initial configuration, a difference was observed between the initial and the optimal configurations, linking the structures between the neighboring dielectric disks. Figure 4 shows a close-up of the link structures. The link structures were regarded as the most important difference between the initial and optimal configurations. The perimeters of the configurations \( L_p \) are shown in the subcaptions in Figs 2(a), 3(a) and 3(b) to investigate the width of the link structures \( L_{\text{link}} \). The difference in the perimeters between the initial and optimal configurations for \( \tau = 1 \times 10^{-6} \) became \( \Delta L_p / L_{\text{design}}^x = 5.65 - 5.11 = 0.54 \). When the width of the link structures in the initial configuration was \( L_p = 0.00 \) and the left and right link structures in each optimal configuration were regarded as the same, the difference in the perimeters \( \Delta L_p \) and \( L_{\text{link}} \) have the following relation:

\[
\Delta L_p \approx 4L_{\text{link}},
\]

and the width of link structures became:

\[
L_{\text{link}} / L_{\text{design}}^x = 1.35 \times 10^{-1} \quad \tau = 1 \times 10^{-6},
\]

\[
L_{\text{link}} / L_{\text{design}}^x = 8.01 \times 10^{-2} \quad \tau = 1 \times 10^{-7}.
\]

Figure 3: (Color online) The optimal configurations, electric field distributions \( E_z / |E_0^i| \) and electric field amplitudes \( |E_z| / |E_0^i| \). The corresponding objective functional values \( F \) are shown in the sub-captions. The relative permittivity values were \( \epsilon_{\text{dm}} = 4.0 \) and \( \epsilon_{\text{air}} = 1.0 \).

The wavelength \( \lambda_{\text{dm}} \) in the dielectric structures \( (\Omega_{\text{dm}}) \) can be written as:

\[
\lambda_{\text{dm}} / L_{\text{design}}^x = 1 / [(\omega L_{\text{design}}^x / 2\pi \epsilon_{\text{dm}}) \sqrt{\epsilon_{\text{dm}}}] = 1.0 \times 10^{-1}.
\]

The wavelength \( \lambda_{\text{dm}} \) and the width of the optimal link structures \( L_{\text{link}} \) were roughly equal to each other. The link structures need to reflect the light to confine the light.
waves in each dielectric disk as a resonator, and they need to transmit the light to the connected circuit. To satisfy both of the above contradictory functions, the widths of the link-structures need to be on the same scale as the wavelength.

Figures 5 and 6 show the frequency and relative permittivity responses of the objective functional value. Periodic steep peaks of the $F$ are observed in Fig. 5 and there are repeated peaks in Fig. 6. When the data shown in Fig. 6 were plotted as the relation between $F$ and $\sqrt{\varepsilon_{\text{dm}}}$, the repeated peaks emerged periodically. The periods in Fig. 5 were $\Delta \left( \frac{\omega L_{\text{design}}}{2\pi c} \right) = 0.296$ and $0.300$ for $\tau = 1 \times 10^{-6}$ and $1 \times 10^{-7}$, respectively. In Fig. 6, $\Delta \left( \sqrt{\varepsilon_{\text{dm}}} \right) = 0.121$ for both $\tau = 1 \times 10^{-6}$ and $1 \times 10^{-7}$.

The circuit length ($L_{\text{circuit}}$) is considered to be the most important length for feedback in the circuit, becoming an integral multiple of the wavelength in a dielectric material as:

$$L_{\text{circuit}} = N \lambda_{\text{dm}},$$

where $N$ is an integer. The above relation can be rewritten as:

$$\frac{\omega L_{\text{design}}}{2\pi c} \sqrt{\varepsilon_{\text{dm}}} = N \frac{L_{\text{design}}}{L_{\text{circuit}}}$$

When difference between integers $N + 1$ and $N$ is taken into consideration, the difference between the oscillating frequencies and the relative permittivity of the dielectric material can be derived as:

$$\Delta \left( \frac{\omega L_{\text{design}}}{2\pi c} \right) = \frac{L_{\text{design}}}{L_{\text{circuit}} \sqrt{\varepsilon_{\text{dm}}}},$$

$$\Delta \left( \sqrt{\varepsilon_{\text{dm}}} \right) = \frac{L_{\text{design}}}{L_{\text{circuit}}} \left( \frac{\omega L_{\text{design}}}{2\pi c} \right)^{-1},$$

which leads to following relations:

$$\frac{L_{\text{Circuit}}}{L_{\text{design}}} = \frac{1}{\Delta \left( \frac{\omega L_{\text{design}}}{2\pi c} \right) \sqrt{\varepsilon_{\text{dm}}}}$$

(5)

$$\frac{L_{\text{Circuit}}}{L_{\text{design}}} = \frac{1}{\Delta \left( \sqrt{\varepsilon_{\text{dm}}} \right)}$$

(6)

The circuit length $L_{\text{Circuit}}$ could be obtained from the values of the periods and the circuit lengths shown in Tables 1 and 2.

<table>
<thead>
<tr>
<th>$\tau$</th>
<th>$\frac{\omega L_{\text{design}}}{2\pi c}$</th>
<th>$\sqrt{\varepsilon_{\text{dm}}}$</th>
<th>$\frac{L_{\text{Circuit}}}{L_{\text{design}}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1 \times 10^{-6}$</td>
<td>0.296</td>
<td>2.0</td>
<td>1.68</td>
</tr>
<tr>
<td>$1 \times 10^{-7}$</td>
<td>0.300</td>
<td>2.0</td>
<td>1.66</td>
</tr>
</tbody>
</table>

Table 2: The values of the period and the circuit length obtained from Fig. 6 and Eq. (6).

<table>
<thead>
<tr>
<th>$\tau$</th>
<th>$\frac{\omega L_{\text{design}}}{2\pi c}$</th>
<th>$\Delta \sqrt{\varepsilon_{\text{dm}}}$</th>
<th>$\frac{L_{\text{Circuit}}}{L_{\text{design}}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1 \times 10^{-6}$</td>
<td>5.0</td>
<td>0.121</td>
<td>1.65</td>
</tr>
<tr>
<td>$1 \times 10^{-7}$</td>
<td>5.0</td>
<td>0.121</td>
<td>1.65</td>
</tr>
</tbody>
</table>

The average $L_{\text{Circuit}}/L_{\text{design}}$ in Tables 1 and 2 is:

$$\frac{L_{\text{Circuit}}}{L_{\text{design}}} = 1.66.$$  

(7)

The perimeter of the disks $L_{\text{disk}}$ is:

$$\frac{L_{\text{disk}}}{L_{\text{design}}} = 2\pi \frac{R_{\text{disk}}}{L_{\text{design}}}$$

(8)

$$= 1.88.$$  

(9)

$L_{\text{Circuit}}$ was nearly the same as $L_{\text{disk}}$. Hence, the resonance in the disk was considered to be the most important feedback mechanism in the WGM resonator circuit.

4. Conclusions

The topology optimization for a WGM resonator circuit based on a level set expression was presented. Clear dielectric boundaries were obtained by linear interpolations of level set functions and the effect of dielectric surfaces was
taken into consideration in the topology optimization process. The link structures between neighboring disks were improved and the light trajectory circuits along the surfaces of the dielectric disks were observed in the optimal configurations. The widths of the link structures were on the scale of the wavelength of light and the widths provided an appropriate rate of reflection and transmission for resonance to occur in the disks, creating a light circuit.

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References